Mathematics for Systems Biology and Bioinformatics

Lecture Prof. Dr. Thomas Filk

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Exercise sheet no. 11

Submission until 23.1.2013 10:00 am in the tutorials

Homework 17: Extended Lotka-Volterra-Model (10 Points)

The original Lotka Volterra System is given by the differential equations

$$\dot{x}(t) = a \cdot x(t) - b \cdot x(t) \cdot y(t) \tag{1}$$
$$\dot{y}(t) = c \cdot x(t) \cdot y(t) - d \cdot y(t) \tag{2}$$

x is the prey population and y is the predator population, all parameters are $\in \mathbb{R}_{>0}$

a) **(2 Points)** Explain the single terms. What can be said about the fixed points of the system? (No calculations, just summarize!)

There are several possible extensions for the original system, here is one:

$$\dot{x}(t) = a \cdot x(t) \cdot \left(1 - \frac{x(t)}{K}\right) - b \cdot \frac{x(t)}{x(t) + S} \cdot y(t) \tag{3}$$
$$\dot{y}(t) = c \cdot \frac{x(t)}{x(t) + S} \cdot y(t) - d \cdot y(t) \tag{4}$$

with $K > 0$ and $S > 0$.

One possible parameter set is $a = b = c = 1$; $d = 1/3$; $K = 30$; $S = 10$. Use these parameters for the following questions.

b) **(2 Points)** What is the meaning of the parameters K and S?
Draw $b\frac{x}{x+S}$ and $ax(1 - \frac{x}{K})$ together with their equivalent in the original model in one graph and explain their influence.

c) **(6 Points)** Show that there are no stable fixed points in the extended model for the given parameter set:

i.) Find the fixed points (x^*, y^*)

ii.) Calculate the Jacobian matrix J and then(!) insert the values of the fixed points (x^*, y^*).

$$J = \left.\begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix}\right|_{(x^*, y^*)} \tag{5}$$

iii.) Calculate the eigenvalues of the Jacobian matrix

Draw a sketch of the behavior of the model near to the fixed point in the phase space.