Mathematics for Systems Biology and Bioinformatics

Lecture Prof. Dr. Thomas Filk

Tutorials Dr. Tim Maiwald, Christian Tönsing

Exercise sheet no. 3

Submission until 14. Nov 2012 10:00 am in the tutorials

Exercise 6: Exponential function

a) Show that for a function, with properties $f'(x) = f(x)$ and $f(x_0 = 0) = 1$, the Taylor series at $x_0 = 0$ is $\sum\limits_{n=0}^{N} \frac{x^n}{n!} = \exp(x)$.

b) Proof that the function $f(x) = \sum\limits_{n=0}^{\infty} \frac{x^n}{n!} = \exp(x)$ repeats itself after differentiation.

Exercise 7: Taylor expansion of $\sin(x)$ and $\cos(x)$

Expand the trigonometric functions $\sin(x)$ and $\cos(x)$ by a Taylor series at $x_0 = 0$. What is the link to the exponential function?

Homework 3: Relations and Functions (6 Points)

A relation $\sim$ is a subset R on a cartesian product:

$$R \subseteq X \times Y \quad with \quad X \times Y := \{(a, b) \mid a \in X \wedge b \in Y\}$$

We name X the domain and Y the range of the relation $\sim$.

a) Relations can be classified by the following terms:

symmetric: $a \sim b \Leftrightarrow b \sim a$
anisymmetric: $(a \sim b \wedge b \sim a) \Rightarrow a = b$
reflexive: $a \sim a$ for all a
total: $a \sim b \vee b \sim a$ for all a, b
transitive: $(a \sim b \wedge b \sim c) \Rightarrow a \sim c$

Fill out the following table with yes or no:

Relation (on $\mathbb{R} \times \mathbb{R}$)	symm.	refl.	antisymm.	total	trans.
$=$					
$\neq$					
$\leq$					
$>$					

b) Sketch the graph of the following relations in a cartesian coordinate system and identify their domain and range.

i.) $q = \{(1, 2), (\frac{1}{2}, 1), (2, 0), (0, -1), (-1, 1)\}$
ii.) $y < 2x$
iii.) $f(x) = x^2$
iv.) $y^2 = x$

c) Which of the following relations is a function and why?

i.) $y^2 = 3x + 2$
ii.) $y < x + 3$
iii.) $s(t) = t^2$
iv.) $g(x) = \frac{1}{\sqrt{x^2+k^2}}$
v.) $q = \{(1, 2), (\frac{1}{2}, 1), (2, 0), (0, -1), (-1, -1)\}$
vi.) $r = \{(1, 2), (2, 1), (0, 1), (-1, -1), (-1, -2)\}$

Homework 4: Harmonic Oscillator: power series (5 Points)

Consider the second order Ordinary Differential Equation (ODE) of the Harmonic Oscillator:

$$\frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} + x(t) = 0 \tag{1}$$

Proof that the solution to this is

$$x(t) = A\,\sin(t) + B\,\cos(t)$$

by using a power series ansatz for $x(t)$:

$$x(t) = \sum_{n=0}^{\infty} a_n t^n.$$

a) Use the power series ansatz in equation (1) an show that

$$\sum_{n=0}^{\infty} \left[a_{n\,+\,2}(n+2)(n+1) + a_n\right] t^n = 0 \tag{2}$$

b) Identify the recurrence relation in equation (2) and analyze it for different values of n, starting from $n = 0$. Split the power series into two separate series, under the condition that each solves the ODE independently (*independent special solutions*).

c) How can the *special solutions* be used to show that the *general solution* is

$$x(t) = A\,\sin(t) + B\,\cos(t)\,?$$

d) Using the latter calculus you can easily write down the approximation of the solution x(t). Test the approximation by drawing the Taylor series up to terms in x^n for n = 1, 6, 10, 20. You may use a graphical calculator, any computer program or the java applet on `http://calculusapplets.com/taylor.html`