Mathematics for Systems Biology and Bioinformatics

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Exercise sheet no. 2

Submission until 7. Nov 2012 10:00 am in the tutorials

Home work 1: Continuous and differentiable functions (5 Points)

Test the following functions for continuity and differentiability. For which values or domains of x is this not the case? What has to be changed to obtain functions according to the definition. If possible, calculate the first derivative and draw the graph of the function within a suitable range.

a) $f(x) = \sqrt{x}$

b) $f(x) = \frac{1}{x^2 - a^2} \ \ , a \in \mathbb{R}$

c) $f(x) = \frac{1}{x^2 + a^2} \ \ , a \in \mathbb{R}$

Home work 2: Taylor series (5 Points)

The Taylor series is a very important tool. It allows to approximate a function f near the value of $x = x_0$ by an polynomial. Let f be a n-times differentiable function and $f^{(k)}(x_0)$ the k-th derivative of f at $x = x_0$. (We denote: $f^{(0)}(x_0) = f(x_0)$). The n-th degree Taylor polynomial of the function f at x_0 is

$$T_{x_o}^n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k.$$

If f is infinitely differentiable at x_0, n gets to infinity. We then call this *Taylor Series*. For a lot of common functions (more precisely for analytical functions) holds

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

for all x for which the infitite sums on the right hand side converge.

Calculate the first 5 derivatives of the funtcion

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

and determine the Taylor polynominal in the neighboruhood of $x_0 = 0$ up to the 5th degree. Draw the first, second and 4th degree Taylor polynominal together with $\cosh(x)$ in one figure. Calculate the relative inaccuracy of the approximation at $x_1 = 0$, $x_2 = 1$, $x_3 = 3$ and $x_4 = 10$.